Self-trapping of optical vortices in waveguide lattices with a self-defocusing nonlinearity

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Abstract: We demonstrate the self-trapping of single- and double-charged optical vortices in waveguide lattices induced with a self-defocusing nonlinearity. Under appropriate conditions, a donut-shaped single-charged vortex evolves into a stable discrete gap vortex soliton, but a double-charged vortex turns into a self-trapped quadrupole-like structure. Spectrum measurement and numerical analysis suggest that the gap vortex soliton does not bifurcate from the edge of the Bloch band, quite different from previously observed gap spatial solitons. Our numerical findings are in good agreement with experimental observations.

OCIS codes: (190.0190) Nonlinear optics; (190.5330) Photorefractive optics; (250.5530) Pulse propagation and temporal solitons

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Optical waves propagating in nonlinear waveguide arrays and photonic lattices have attracted a great deal of interest [1, 2]. One of the key paradigms of discretizing light behavior in periodical structures is the self-trapped states better known as “lattice solitons” [3-8]. Such discrete spatial solitons typically have their propagation constants residing in the semi-infinite gap (arising from the total internal reflection) or inside a true photonic band gap (arising from the Bragg reflection). Although gap solitons were traditionally considered as a temporal phenomenon in one-dimensional (1D) periodic media, spatial gap solitons in both 1D and 2D configurations have been demonstrated recently in a number of experiments with either a self-focusing or a self-defocusing nonlinearity [5, 6, 9-12].

Optical vortex solitons propagating in periodic media form another family of discrete spatial solitons with helical phase structures, as have been predicted earlier [13, 14] and demonstrated in optical induced lattices with a self-focusing nonlinearity [15, 16]. These are semi-infinite-gap vortex solitons. Gap vortex solitons with their propagation constants located inside a photonic bandgap have only been observed so far with a self-focusing nonlinearity where the vortex solitons bifurcate from the band edge of the second band, thus named as second-band vortex solitons [17]. To our knowledge, gap vortex solitons in self-defocusing lattices have never been demonstrated, although they have been predicted in theory to exist as spatially localized topological states in Bose-Einstein Condensates confined by an optical lattice [18] as well as in photorefractive optically-induced photonic lattice [19, 20].

In this paper we report the first experimental demonstration of self-trapping of both single- and double-charged vortex beams by on-axis excitation in a “backbone” photonic lattice induced with a saturable self-defocusing nonlinearity. We show that, under proper nonlinear conditions, a single-charged (m=1) vortex beam can evolve into a gap vortex soliton, while a double-charged (m=2) vortex beam tends to turn into a self-trapped quadrupole-like structure. The spatial power spectra and interferograms (with a tilted plane wave) of the self-trapped vortices from both experiments and numerical simulations are presented, and the stability of the vortex solitons is also studied numerically. Our results show that the gap vortex soliton does not bifurcate from the edge of the first Bloch band, quite different from all previously observed fundamental, dipole or quadrupole spatial gap solitons.

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The experimental setup for our study is similar to those used earlier for observation of discrete (semi-infinite gap) vortex solitons in self-focusing lattices [15], except that we now use a self-defocusing nonlinearity to induce the waveguide lattices [12, 21]. The lattice is induced in a photorefractive SBN crystal (5×10×5 mm^3) by a spatially modulated partially coherent light beam sent through an amplitude mask. The mask is appropriately imaged onto the input face of the crystal, creating a periodic input intensity pattern for lattice induction. The lattice period is about 27 μm. With a negative bias voltage, the intensity pattern induces a "backbone" waveguide lattice, as the crystal turns into a defocusing nonlinear medium [22]. The vortex beam is generated by sending a coherent laser beam through a computer generated vortex hologram. In all experiments, the lattice beam is ordinarily-polarized while the vortex beam is extraordinarily-polarized. Thus the lattice beam will undergo nearly linear propagation in the crystal while the vortex beam will experience a large nonlinearity due to the anisotropic property of the photorefractive crystal [5-8]. An incoherent white light source was used as a background illumination to fine tune the screening nonlinearity. The output beam patterns and Fourier spectra are monitored with CCD cameras. The vortex beam exiting the crystal is also sent into a Mach-Zehnder interferometer for phase measurement as needed.

In our experiment, the off-site excitation scheme is used so that the vortex core is on an index minimum while the donut-like vortex beam covers four adjacent index maxima. To open the first Bragg reflection gap (between the first and second Bloch bands), a relatively high lattice beam intensity and bias field is employed for induction of a deep lattice potential [12, 21]. By fine-tuning the nonlinearity (through the bias field and the lattice-to-background intensity ratio), self-trapping of the vortices can be established. Typical experimental results are presented in Fig. 1, for which the intensity ratio of the vortex beam to the lattice beam is about 1:4, and the bias field is about -1.2 kV/cm. The interferograms of the input vortex beams with a tilted plane wave are shown in Fig. 1(a), where the central fork resulting from the fringe bifurcation indicates the phase singularity (m=1 for top panels, and m=2 for bottom panels) of the vortex beam. When self-trapping is established in the nonlinear regime, both m=1 and m=2 vortices assume an intensity pattern primarily consisting of four spots [Fig. 1(b)], similar to the semi-infinite-gap vortex solitons [15, 16]. Along the directions of the principal axes of the square lattice (which are oriented diagonally rather than horizontally and vertically), long "tails" beyond the central four spots can be seen. Although the intensity patterns of self-trapped m=1 and m=2 vortices look somewhat similar, significant differences can be found in their phase structure and spatial spectrum.

First, we use two different interference techniques to identify the phase structure of self-trapped vortices as used earlier for vortices in self-focusing lattices [23]. One is to send a tilted broad beam (quasi-plane wave) to interfere with the output vortex beam [Fig. 1(c)]. For the limited propagation distance of our crystal length (10 mm), it appears that the vortex singularity (manifested by the central fork in the interferograms) persists after the nonlinear propagation through the crystal, although it seems that charge-flipping (reversal of forks) is associated with the m=2 but not the m=1 vortices at the crystal output. However, as shown below from numerical simulations to longer propagation distance, the singularity can maintain only for the m=1 but not for the m=2 vortices. In fact, our theoretical analysis shows that a "true" double-charged gap vortex soliton does not exist under this excitation condition, and a quadrupole-like soliton structure is found instead for the m=2 vortex. The other technique is to send a co-axial broad Gaussian beam as an interfering beam. We can see clearly that the phase structures for self-trapped m=1 and m=2 vortices are different [Fig.1 (d)]. The two diagonal spots are out-of-phase for the m=1 vortex but in-phase for the m=2 vortex, similar to self-trapped vortices in self-focusing photonic lattices [15, 16, 23].

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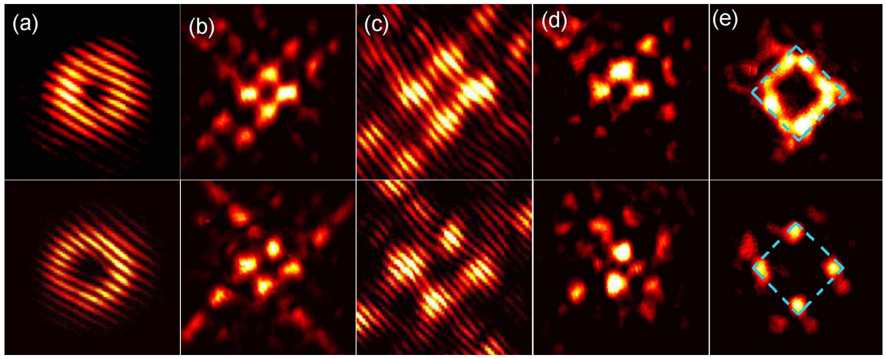

Fig. 1. Experimental results of self-trapping of single-charged (top) and double-charged (bottom) vortices in a defocusing photonic lattice. (a) Interferograms showing the phase of the input vortex beams, (b) intensity patterns of self-trapped vortex beams at lattice output, (c, d) interferograms between (b) and a tilted plane wave (c) and an on-axis Gaussian beam (d), respectively, and (e) the k-space spectra of (b) where the dash squares mark the first Brillouin-zone of the square lattice. (a, c) are zoomed in with respect to (b, d) for better visualization.

Next, we measure the spatial spectrum of self-trapped vortices [Fig. 1(e)] by using the technique of Brillouin Zone (BZ) Spectroscopy [24]. Again, dramatic differences between m=1 and m=2 vortices can be seen in these k-space spectra, indicating quite different physical pictures for self-trapping. For the m=1 vortex, most of the power is located alongside the first BZ, but it would not concentrate just to the four corner points (corresponding to four high-symmetry M points) which mark the edge of the first Bloch band and where the diffraction is anomalous [6, 25]. For the m=2 vortex, however, the nonlinear spectrum reshaping makes the power spectrum settle into the M-points quickly, similar to those of the fundamental gap solitons and gap soliton trains [12]. Numerical simulations (see below) show that such spectrum difference remains for long propagation distance. These results suggest that although the m=1 vortex can evolve into a gap vortex soliton, it does not bifurcate from the edge of the first Bloch band, quite different from all previously observed fundamental, dipole or quadrupole-like gap solitons in self-defocusing lattices [6, 12, 21]. It is also significantly different from the second-band gap vortex solitons [17] or the reduced-symmetry gap solitons [26] in self-focusing lattices, which all bifurcate from the edge of the second band. On the other hand, the m=2 vortex can evolve into a quadrupole-like localized state, which does seem to bifurcate from the edge of the first Bloch band as confirmed by numerical analysis below. We would like to mention that in Fig. 1 we did not show the linear output of the vortex beams simply due to that the linear output pattern does not differ significantly as compared to the nonlinear output of Fig. 1(b) in our experiment. This is because the induced lattice potential is deep (for opening the first gap [12, 21]) and the length of our photorefractive crystal is only 10 mm so the vortex beam does not exhibit strong discrete diffraction as clearly seen in our simulations to longer propagation distances. However, the experimentally measured phase and spectrum of the linear output are apparently different from those of nonlinear output. In the linear region, all adjacent intensity peaks from the vortex beam have an in-phase relationship and the power spectrum covers the entire first BZ with most of the energy concentrated in the center of the BZ.

We now compare the above experimental observations with our numerical results obtained using beam propagation simulations with the initial condition similar to that for the experiment. The numerical model is a nonlinear wave equation with a 2D square lattice potential under self-defocusing photorefractive nonlinearity [6, 27]. Figure 2 shows the typical simulation results corresponding to experimental results of Fig. 1. Excellent agreement can be seen for the 10 mm of propagation distance (i.e. our crystal length) for both m=1 and m=2 vortices. In particular, even for only 10 mm of propagation, clear differences can be seen in

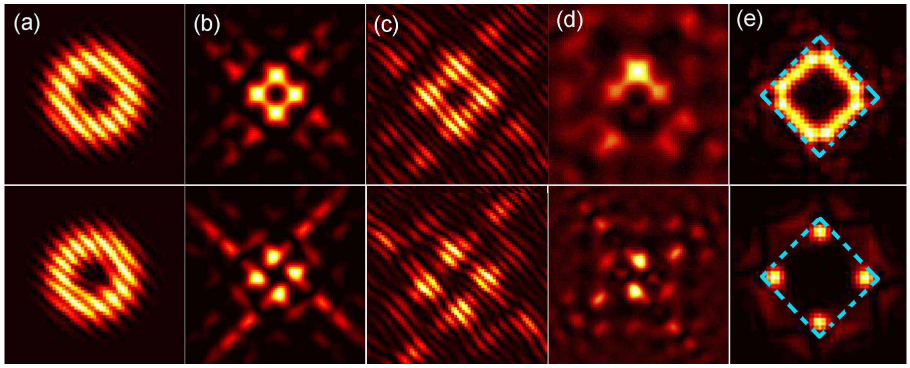

Fig. 2. Numerical results of self-trapping of single-charged (top) and double-charged (bottom) vortices in a defocusing photonic lattice corresponding to experimental results of Fig. 1. The propagation distance is 10 mm corresponding to the length of the crystal used in experiment.

the phase structure [Fig. 2(c, d)] and k-space power spectrum [Fig. 2(e)], as observed in our experiments (Fig. 1). To examine whether the relevant gap soliton structures can persist for longer propagation distances, simulations are also performed with a propagation distance up to 40 mm while all other parameters are left unchanged. The results are shown in the left panels of Fig. 3. Indeed, the corresponding intensity patterns are found to be nearly unchanged even after 40 mm of propagation. However, by interfering the vortex beam with a titled plane wave to observe the phase structure, a major difference is noticed after 40-mm of propagation: while the fork is still in the center of the interferogram for the m=1 vortex (hence showing that the m=1 gap vortex can maintain its helical phase structure), this is not the case for the m=2 vortex. In the latter case, the forks in the center disappear gradually and the vorticity is eventually lost. In fact, the m=2 vortex loses its original angular momentum and transforms itself into a quadrupole-like structure. Before the vorticity completely disappears, a transient state of charge flipping is found from our detailed simulations, but unlike the periodical appearance of m=2 and m=-2 vortices found in the self-focusing case [23], the m=2 vortex singularity cannot maintain in self-defocusing lattices, and the vortex disintegrate into an unstable quadrupole-like structure. (In the self-focusing case, the quadrupole appears only as a transient state for charge flipping of the m=2 vortex under the isotropic photorefractive lattice potential [23]). This dynamical evolution can be seen more clearly in the 3D plot of beam propagation illustrated in the right panels of Fig. 3. Furthermore, our numerical simulations to longer propagation distance also indicate that the tails of the self-trapped m=2 vortex have wave properties typical to Bloch modes located in the vicinity of the first-band M point (being out-of-phase between adjacent sites along directions of the lattice principal axes [25]). This is consistent with the k-space power spectrum that settles onto four M points, indicating that the m=2 vortex evolves into a gap quadrupole soliton bifurcated from the edge of the first Bloch band. On the other hand, similar simulations to 40-mm propagation distance for the m=1 vortex does not show this well-defined phase relation in the tails [Fig. 3(a)], as some neighboring sites are in-phase and some are out-of-phase along directions of the lattice principal axes. The power spectrum concentrates more into the four sides of the first BZ rather than evolves into a well-defined four M-point spectrum as in Fig. 3(b), suggesting that the m=1 gap vortex soliton does not bifurcate from the edge of the first Bloch band.

Finally, we investigate the stability of both m=1 and m=2 self-trapped vortices by means of linear stability analysis for typical parameters corresponding to experimental observations. Our analysis shows that, indeed, the m=1 vortex gap soliton is stable almost throughout the first gap of the defocusing lattice, while the quadrupole gap state is always linearly unstable. Since the latter instability growth rate is relatively small, the quadrupolar structure is observable for certain propagation distances, as demonstrated in our experiment and numerical simulations. The soliton solutions (in real and Fourier space) and the corresponding

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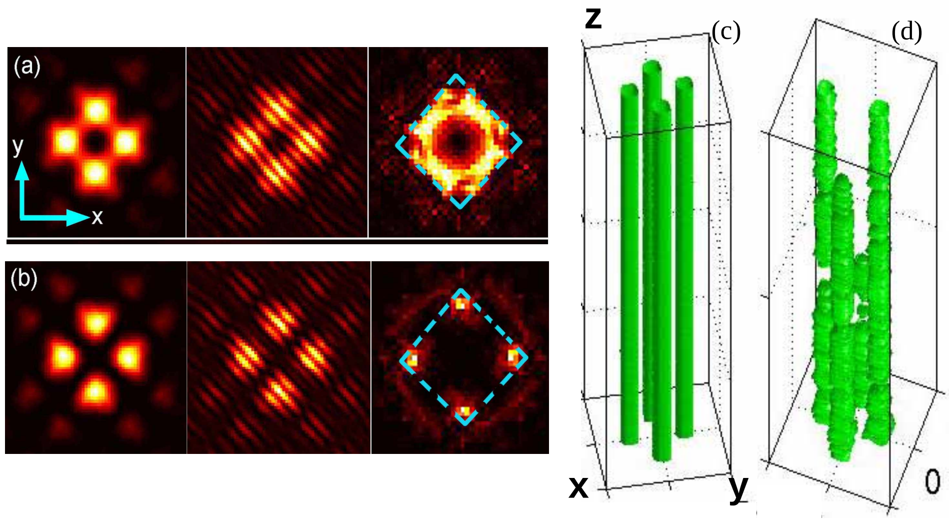

Fig. 3. Simulation results of single-charged (a) and double-charged (b) vortex beams propagating to a longer distance of 40 mm. Left panels show the output transverse (x-y) intensity pattern (left), its interferogram with a titled plane wave (middle), and its k-space spectrum (right) in both (a) and (b). Notice that the vortex singularity maintains in (a) but disappears in (b). Right panels show the propagation of a stable m=1vortex beam (c) and of an unstable quadrupole beam as arising from the breakup of m=2 vortex (d) long the longitudinal z-direction (from bottom to top) through the defocusing lattice.

maximal growth rates [maximum real part Re(λ) of the linearization eigenvalues] as a function of the propagation constant μ are illustrated in Fig 4, where regions of zero growth rate (Max[Re(λ)]=0) indicate the good stability of the gap soliton solutions. We note again that, as seen in Fig. 4 (right panels), while the unstable quadrupolar structure seems to bifurcate from a linear Bloch mode of the first band, the same is not true for the m=1 gap vortex soliton, as the latter stability (and corresponding existence) curve appears to have a turning point before reaching the band edge. Results from our experimental observation and numerical analysis are in good agreement with recent theoretical work on the families of the m=1 gap vortex solitons in periodic media [28], where it also shows that the single-charged vortex families do not bifurcate from edges of Bloch bands, but rather they turn back and move into band gaps before reaching band edges. Although such non-edge bifurcation of vortex gap solitons can be found from the mathematical model of nonlinear propagation of vortices in 2D periodic media, it seems that the underlying physical mechanism for the emergence of such "purely nonlinear" states merits further investigation. Intuitively, this might be attributed to the nontrivial helical phase structure of the vortex, which cannot be expressed as a simple superposition of linear Bloch modes near the band edge. A related interesting issue is to explore the existence and maybe to experimentally demonstrate the genuine m=2 gap vortex solitons with different excitation conditions, such as those gap vortex states proposed in [29]. Although the gap vortices in periodic structures have been previously proposed in literatures [18, 28, 30], the theoretical analysis presented here with a model involving a saturable self-defocusing nonlinearity is particularly relevant to our experiments and helpful for a good understanding of the experimental observations. Furthermore, the features of the nonlinear spectrum reshaping and instability analysis which have not been illustrated before will stimulate further theoretical study of spatial gap vortex solitons in periodic systems beyond optics.

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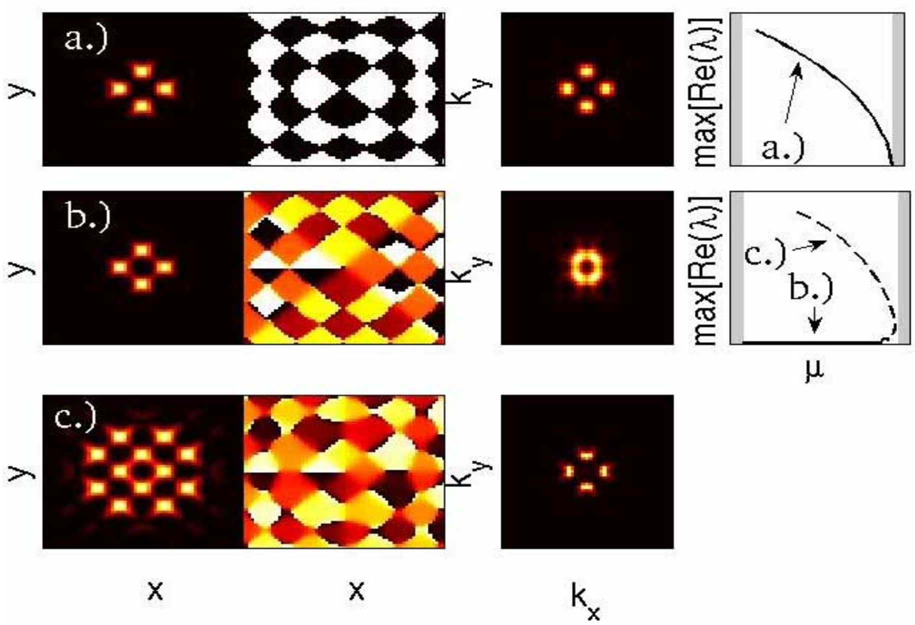

Fig 4. Numerical solutions of self-trapped quadrupole [a.)] and single-charged vortices [b.) and c.)]. Shown are typical stationary patterns (first column), corresponding phase structure (second column), Fourier spectra (third column), and maximal instability growth rates (fourth column). Plots in fourth column are given as a function of the propagation constant μ, while the spectral bands are denoted by shaded areas. The first Bloch band is located to the right, where for the single-charged vortex family the stable node branch [solid line, b.)] collides with the unstable saddle branch [dashed line, c.)] before reaching the band edge. Zero growth rates indicate that the self-trapped structure is linearly stable.

In conclusion, we have demonstrated self-trapping of both single- and double-charged vortex beams in optically induced photonic lattices with a self-defocusing nonlinearity. We have shown that only the single-charged vortex beam can evolve into a gap vortex soliton which does not bifurcate from the band edge, while the double-charged vortex eventually reshapes into a quadrupole-like gap soliton which does bifurcate from the edge of the Bloch band. The single-charged vortex gap soliton can be linearly stable under certain conditions while the double-charged vortex tends to break up into a quadrupole-like structure which is linearly unstable. Our experimental results are corroborated by numerical simulations.

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